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Observer-Based Hybrid Fuzzy Controller Design and Its Application to Two-Wheeled Self-Balancing Robot

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Abstract: An observer-based hybrid fuzzy controller (OBHFC) for a certain class of unknown nonlinear dynamical system is developed in this study. The proposed OBHFC is designed according to the transparency of the plant knowledge and the control information. Further, to confront the lack of state information, a state observer is added to estimate the tracking error vector, and then the states can be obtained completely. Utilization of the observer-based output feedback control, the controller's parameters can be tuned in the sense of Lyapunov stability theorem. Finally, the proposed OBHFC is extended its application to balance control a two-wheeled robot. In the balance controller, the software observer is embedded to replace the gyroscope to reduce the hardware cost. To verify its effectiveness, some simulations are carried out. The performances of simulated data are compared with conventional adaptive fuzzy sliding-mode controller (AFSMC). From these results, the proposed control scheme possesses some features of stable tracking performance, lower hardware cost and shorter response time.

Keywords: Adaptive Law; Balance Control; Fuzzy Control; State Observer; Two-Wheeled Robot.

Introduction

Many physical systems (e.g., wheeled or biped robot, micro-electro-mechanical system, aircraft control, biomedical engineering, etc.) are nonlinear and dynamical. Due to the highly nonlinearity, uncertain characteristics and incomplete information within them, it is difficult to evaluate the appropriate control effort to track the desired trajectory. To this end, much research has been done to apply various approaches to tune the control effort via adaptive algorithm, the predicted uncertainty or observed state information [1-10]. However, if the adaptation parameters are not properly chosen, the tracking response or the control effort will be unsatisfactory when large parameter variations or external load disturbances occur. Furthermore, it is difficult to get the complete information of uncertainties or system states in practical application. Therefore, the first aim of this study is to design an alternative and effective control scheme to confront the problem existed in practical environments of nonlinear dynamical system. On the application aspect, two-wheeled self-balancing robot has been finding wide application in personal transporter or data acquisition; however, it has highly nonlinear dynamical characteristics.

In the past four decades, fuzzy systems have supplanted conventional technologies in many applications, especially in fuzzy control (FC). Based on the universal approximation theorem [11-13], many adaptive fuzzy sliding-mode controllers (AFSMC) have been proposed to incorporate with the expert information systematically and the stability can be guaranteed by theoretical analysis [3, 5, 6, 14]. On the tuning aspect, various adaptive algorithms, uncertainty predictors or state observers have been enunciated [15-20]. One major advantage of these schemes is that the adaptive laws are derived based on the Lyapunov synthesis method and, therefore, the stability of the controlled systems could be guaranteed. However, if the plant knowledge or control information is incomplete seriously then the load of adaptive algorithm will be heavy, the response time will be lengthened and the tracking error will be large. On the practical aspect, the working environment of many physical systems may be wicked and the information is unable to know in advance. For example, the encountered environment of a two-wheeled self-balancing robot may be a bumpy road [21-25]; while the amount of provided information is related with the number of utilized sensors which will influence the robot cost. So it is a stringent topic to research a robust balance controller for the two-wheeled self-balancing robot or other certain class of unknown nonlinear dynamical systems.

To accomplish the mentioned motivation, an observer-based hybrid fuzzy controller (OBHFC) is developed in this paper. According to the transparency of the plant knowledge and the control information, an adaptive hybrid fuzzy controller (AHFC) is designed as the main controller. To confront the lack of state information in practical applications, a state observer is added to estimate the states, and then the parameters of the AHFC can be tuned in the sense of Lyapunov stability theorem. Moreover, in order to reduce the hardware cost, this study attempts to extend the application of the OBHFC to balance a two-wheeled robot.

T-S Fuzzy Model Control System

Consider a nth-order nonlinear dynamical system of the form

$$x^{(n)} = f(\boldsymbol{x}) + g(\boldsymbol{x})u + d(t) \\ y = x \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \dots, x_n]^T = [x, \dot{x}, \dots, x^{(n-1)}]^T \in R^n$ is the state vector of the system; $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are unknown but bounded continuous functions of the nonlinear system; $u \in R$ and $y \in R$ are the control effort and output of the system, respectively; $d(t)$ denotes the bounded external disturbance. Here, assume that not all states x_i are assumed to be available for measurement, only the system output y is assumed to be measurable, hence states observer $\hat{\boldsymbol{x}}$ will be designed to estimate the nonlinear system state. In order for (1) to be controllable, it is required that $0 < g(\boldsymbol{x}) < \infty$ for $\boldsymbol{x}$ in certain controllability region Un_x and also it is assumed that $d(t)$ have upper bound D; that is, $|d(t)| \le D$. Without the loss of generality, the following assumption is made:

Assumption 1:

$\boldsymbol{x}$ belongs to compact set U_x. Also, $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are bounded as $|f(\boldsymbol{x})| \le F$ and $0 \le g_L \le g(\boldsymbol{x}) \le g^U$, respectively.

In state-space representation, Equation (1) can be rewritten as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}[f(\boldsymbol{x}) + g(\boldsymbol{x})u + d(t)] \\ y = \boldsymbol{C}^T\boldsymbol{x} \tag{2}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Now, the fuzzy model is described by fuzzy IF-THEN rules and will be employed here to deal with the control design problem for the nonlinear system. Since the consequent part of T-S fuzzy model is a linear equation rather than fuzzy sets or constant values, this model can be viewed as a somewhat piecewise linear function, where the change from one piece to another is smooth rather than abrupt. However, $f(\boldsymbol{x})$ and u are unknown, so estimated $\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f)$ and $u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_u)$ are applied, respectively.

Plant Fuzzy Rules

The lth rule of $\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f)$ is of the following form:

$$R_{\hat{f}}^{l}: \text{IF } \hat{x}_1 \text{ is } A_1^l \text{ and } \cdots \text{ and } \hat{x}_n \text{ is } A_n^l, \\ \text{THEN } \hat{f}^{\,l} = a_0^l + a_1^l\hat{x}_1 + \cdots + a_n^l\hat{x}_n \tag{3}$$

where $\hat{\boldsymbol{x}} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]^T$ is the estimated state vector; the lth θ_f^T is represented as $\boldsymbol{\theta}_f^{l\,T} = [a_0^l, a_1^l, \dots, a_n^l]$; $l = 1,2,\dots,M$, M is the number of the fuzzy IF-THEN rules; A_i^l are fuzzy sets of $\hat{x}_i (i = 1,2,\dots,n)$. In this study, the Gaussian functions, singleton fuzzifier, product inference and center-average defuzzifier method are adopted. Then, $f(\boldsymbol{x})$ can be estimated by fuzzy logic inference mechanism as follows:

$$\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f) = \frac{\sum_{l=1}^{M} \hat{f}^{\,l} \cdot [\prod_{i=1}^{n} A_i^l(\hat{x}_i)]}{\sum_{l=1}^{M} [\prod_{i=1}^{n} A_i^l(\hat{x}_i)]} \\ = \frac{\sum_{l=1}^{M} \boldsymbol{\theta}_f^{l\,T} [1\ \hat{\boldsymbol{x}}^T]^T \cdot [\prod_{i=1}^{n} A_i^l(\hat{x}_i)]}{\sum_{l=1}^{M} [\prod_{i=1}^{n} A_i^l(\hat{x}_i)]} \tag{4}$$

where $[1\hat{\boldsymbol{x}}^T]^T = [1, \hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]^T$; $A_i^l(\hat{x}_i)$ is membership function value of the fuzzy variable $\hat{x}_i$ in A_i^l. Equation (4) can be simplified as

$$\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f) = \boldsymbol{\theta}_f^T \boldsymbol{\xi}_f(\hat{\boldsymbol{x}})$$

where $[1\hat{\boldsymbol{x}}^T]^T = [1, \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$; $A_i^l(\hat{x}_i)$ is membership function value of the fuzzy variable $\hat{x}_i$ in A_i^l. Equation (4) can be simplified as

$$\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f) = \boldsymbol{\theta}_f^T \boldsymbol{\xi}_f(\hat{\boldsymbol{x}}) \tag{5}$$

where $\boldsymbol{\theta}_f^T = \left[\boldsymbol{\theta}_f^{1\,T}, \boldsymbol{\theta}_f^{2\,T}, \boldsymbol{\theta}_f^{3\,T}, \ldots, \boldsymbol{\theta}_f^{M\,T}\right]$ and $\boldsymbol{\xi}_f(\hat{\boldsymbol{x}}) = [\boldsymbol{\xi}_f^1(\hat{\boldsymbol{x}}), \boldsymbol{\xi}_f^2(\hat{\boldsymbol{x}}), \ldots, \boldsymbol{\xi}_f^M(\hat{\boldsymbol{x}})]^T$ are adjustable parameter vector and fuzzy basic function vector, respectively. $\boldsymbol{\xi}_f^l(\hat{\boldsymbol{x}})$ is defined as

$$\boldsymbol{\xi}_f^l(\hat{\boldsymbol{x}}) = \frac{[1\ \hat{\boldsymbol{x}}^T]^T [\prod_{i=1}^n A_i^l(\hat{x}_i)]}{\sum_{l=1}^M [\prod_{i=1}^n A_i^l(\hat{x}_i)]} \tag{6}$$

Control Fuzzy Rules

The lth rule of $u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_u)$ is of the following form:

$$R_{u_D}^l: \text{IF } \hat{x}_1 \text{ is } B_1^l \text{ and } \cdots \text{ and } \hat{x}_n \text{ is } B_n^l, \quad \text{THEN } u_D^l = b_0^l + b_1^l \hat{x}_1 + \cdots + b_n^l \hat{x}_n \tag{7}$$

where the lth ${\theta_u}^T$ is represented as ${\boldsymbol{\theta}_u^l}^T = [b_0^l, b_1^l, \ldots, b_n^l]$; B_i^l are fuzzy sets of $\hat{x}_i (i = 1,2,\ldots,n)$. Similarly, the singleton fuzzifier, product inference and center-average defuzzifier method are adopted, Then, the output, $u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_u)$, can be obtained by fuzzy logic inference mechanism as follows:

$$u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_u) = \frac{\sum_{l=1}^M u_D^l \cdot [\prod_{i=1}^n B_i^l(\hat{x}_i)]}{\sum_{l=1}^M [\prod_{i=1}^n B_i^l(\hat{x}_i)]} = \frac{\sum_{l=1}^M {\boldsymbol{\theta}_u^l}^T [1\ \hat{\boldsymbol{x}}^T]^T \cdot [\prod_{i=1}^n B_i^l(\hat{x}_i)]}{\sum_{l=1}^M [\prod_{i=1}^n B_i^l(\hat{x}_i)]} \tag{8}$$

where $B_i^l(\hat{x}_i)$ is membership function value of the fuzzy variable $\hat{x}_i$ in B_i^l. Equation (8) can be simplified as

$$u_D(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_u) = \boldsymbol{\theta}_u^T \boldsymbol{\xi}_u(\hat{\boldsymbol{x}}) \tag{9}$$

where $\boldsymbol{\theta}_u^T = [\boldsymbol{\theta}_u^{1\,T}, \boldsymbol{\theta}_u^{2\,T}, \boldsymbol{\theta}_u^{3\,T}, \ldots, \boldsymbol{\theta}_u^{M\,T}]$ and $\boldsymbol{\xi}_u(\hat{\boldsymbol{x}}) = [\boldsymbol{\xi}_u^1(\hat{\boldsymbol{x}}), \boldsymbol{\xi}_u^2(\hat{\boldsymbol{x}}), \ldots, \boldsymbol{\xi}_u^M(\hat{\boldsymbol{x}})]^T$ are adjustable parameter vector and fuzzy basic function vector, respectively. $\boldsymbol{\xi}_u^l(\hat{\boldsymbol{x}})$ is defined as

$$\boldsymbol{\xi}_u^l(\hat{\boldsymbol{x}}) = \frac{[1\ \hat{\boldsymbol{x}}^T]^T [\prod_{i=1}^n B_i^l(\hat{x}_i)]}{\sum_{l=1}^M [\prod_{i=1}^n B_i^l(\hat{x}_i)]} \tag{10}$$

Assumption 2: $\hat{\boldsymbol{x}}$ belgs to compact set $U_{\hat{\boldsymbol{x}}}$. Set $\boldsymbol{\theta}_f \in \Omega_f$ and $\boldsymbol{\theta}_u \in \Omega_u$, $\Omega_f = \{\boldsymbol{\theta}_f \in \Re^M : \|\boldsymbol{\theta}_f\| \le M_{\boldsymbol{\theta}_f}\}$, $|\hat{f}(\hat{\boldsymbol{x}}|\boldsymbol{\theta}_f)| \le F$ and $\Omega_u = \{\boldsymbol{\theta}_u \in \Re^M : \|\boldsymbol{\theta}_u\| \le M_{\boldsymbol{\theta}_u}\}$.

Design of Observer-Based Hybrid Fuzzy Controller

Consider an *n*th-order nonlinear dynamical system (1), the system output y to be measurable and the desired output y_r, assume $y_r, \dot{y}_r, \ldots, {y_r}^{(n-1)}$ to be bounded and provided. Because not all states of the nonlinear system are available completely, hence an OBHFC is proposed to search the suitable control effort $u(t)$ so that the actual output $y(t)$ can asymptotically approach the desired output $y_r(t)$. According to the transparency of the plant knowledge and the control information, an AHFC is designed as the main controller. To confront the lack of state information in practical applications, a state observer is added to estimate the tracking error vector, and then the parameters of the AHFC can be tuned in the sense of Lyapunov stability theorem.

Design of Hybrid Fuzzy Controller

To begin with, the desired output vector $\boldsymbol{y}_r$, tracking-error vector $\boldsymbol{e}$ and estimated tracking-error vector $\hat{\boldsymbol{e}}$are defined as $\boldsymbol{y}_r = [y_r, \dot{y}_r, \ldots, {y_r}^{(n-1)}]^T$, $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{y}_r = [e, \dot{e}, \ldots, e^{(n-1)}]^T$ and $\hat{\boldsymbol{e}} = \hat{\boldsymbol{x}} - \boldsymbol{y}_r = [\hat{e}, \dot{\hat{e}}, \ldots, \hat{e}^{(n-1)}]^T$, respectively. The vectors $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{e}}$ denote the estimates of $\boldsymbol{x}$ and $\boldsymbol{e}$, respectively. Then a sliding surface H is defined as

$$H : \{\boldsymbol{e} | S(\boldsymbol{e}) = \boldsymbol{a}^T \boldsymbol{e} = 0\} \tag{11}$$

where $\boldsymbol{a} = [a_1, a_2, \ldots, a_n]^T$ is chosen such that $a_n = 1$ and all the roots of polynomial $s^{n-1} + a_{n-1}s^{n-2} + \cdots + a_2 s + a_1 = 0$ are in the open left-half plane, where *s* is the Laplace operator. The control objective is achieved by finding a suitable control law satisfying the sliding condition $S\dot{S} \le -\rho|S|$, $\rho > 0$.

Based on the universal approximation theorem [11-13], the fuzzy system $\hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f)$ in (4) is capable of uniformly approximating any well-defined nonlinear function $f(\boldsymbol{x})$ over a compact set U_x and an equivalent controller can be obtained. However, if unpredictable disturbances from the parameter variations or external load disturbance occur, the equivalent control effort cannot ensure the favorable control performance. Thus, an auxiliary hitting control effort should be added to eliminate the effect of the unpredictable disturbances. Totally, the control law can be represented as

$$u_S = \frac{1}{g(\boldsymbol{x})}\left(-\sum_{i=1}^{n-1} a_i \hat{e}^{(i)} - \hat{f}(\hat{\boldsymbol{x}} \mid \boldsymbol{\theta}_f) + y_r^{(n)}\right) - \beta \cdot \tanh(\gamma \hat{S}) \tag{12}$$

where $\beta \ge g_L^{-1}(\rho + D)$ is the hitting control gain; γ is a positive constant and $\hat{S} = \boldsymbol{a}^T \hat{\boldsymbol{e}}$. By this method, the alleviation of chattering phenomenon and a smooth control output can be achieved. Furthermore, the derived control law in (12) can satisfy following sliding condition:

$$\hat{S}\ \dot{\hat{S}} = \hat{S}[a_1\dot{\hat{e}} + a_2\ddot{\hat{e}} + \cdots + a_{n-1}\hat{e}^{(n-1)} + \hat{e}^{(n)}]$$

$$= \hat{S}[\sum_{i=1}^{n-1} a_i\hat{e}^{(i)} + \hat{f}(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_f) + g(\boldsymbol{x}) \quad (13)$$

$$+d - y_r^{(n)}] \qquad \leq -\rho|\hat{S}|$$

If the control information is more reliable and more important than the given plant knowledge, it is very helpful to bring in the conventional AFSMC law (Equation (9)) to further promote the control performance. So an enhanced hybrid control mode is proposed in this study according to the transparency of the plant knowledge and the control information. That is, a good choice of the final controller is a weighted sum of u_S and u_D, namely, the final controller is

$$u = \alpha u_S + (1-\alpha)u_D \quad (14)$$

where $\alpha \in [0,1]$ is a hybrid factor. It will be adjusted by the given plant knowledge or the control information. If the control information is more reliable and more important than the given plant knowledge, a smaller α should be chosen; otherwise, a larger α should be chosen.

Design of State Observer

Because not all system states are available completely, hence a state observer must be added to estimate the system information, i.e. the estimated state vector, $\hat{\boldsymbol{x}} = [\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n]^T$, must be provided immediately. After some substitution manipulations, the close-loop dynamical equation of system (1) is obtained as

$$e^{(n)} = -\boldsymbol{a}_m^T\hat{\boldsymbol{e}} + g(\boldsymbol{x})\{-\alpha\beta\tanh(\gamma\hat{S}) + (1-\alpha)$$

$$[u_D(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_u) - \hat{u}_{eq}]\} + f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_f) + d \quad (15)$$

where $\boldsymbol{a}_m = [0, a_1, \dots, a_{n-1}]^T$. Equation (15) can be modified as the error state form

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{a}_m^T\hat{\boldsymbol{e}} + \boldsymbol{B}\{g(\boldsymbol{x})[-\alpha\beta\tanh(\gamma\hat{S})$$

$$+(1-\alpha)(u_D(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_u) - \hat{u}_{eq})] \quad + f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_f) + d\}$$

$$e_1 = \boldsymbol{C}^T\boldsymbol{e} \quad (16)$$

where $e_1 = y - y_r = x_1 - y_r$ denotes the output tracking error. According to (16), the state observer is designed as

$$\dot{\hat{\boldsymbol{e}}} = \boldsymbol{A}\hat{\boldsymbol{e}} - \boldsymbol{B}\boldsymbol{a}_m^T\hat{\boldsymbol{e}} + \boldsymbol{k}_o(\hat{e}_1 - e_1) - \boldsymbol{B}g(\boldsymbol{x})\alpha\beta_{\tanh}(\gamma\hat{S})$$

$$\hat{e}_1 = \boldsymbol{C}^T\hat{\boldsymbol{e}} \quad (17)$$

where $\boldsymbol{k}_o = [k_n, k_{n-1}, \dots, k_1]^T$ is a designed observer gain vector. The design steps of $\boldsymbol{k}_o$ are described as follows. Firstly, the observation errors are defined as $\tilde{e}_1 = \hat{e}_1 - e_1 = \hat{x}_1 - x_1$ and $\tilde{\boldsymbol{e}} = \hat{\boldsymbol{e}} - \boldsymbol{e} = \hat{\boldsymbol{x}} - \boldsymbol{x}$. After subtracting (16) from (17), the observation error dynamical equation is

$$\dot{\tilde{\boldsymbol{e}}} = \boldsymbol{A}\tilde{\boldsymbol{e}} + \boldsymbol{k}_o\boldsymbol{C}^T\tilde{\boldsymbol{e}} - \boldsymbol{B}\{g(\boldsymbol{x})(1-\alpha)[u_D(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_u) - \hat{u}_{eq}] \ + f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_f) + d\}$$

$$= \boldsymbol{\Lambda}_o\tilde{\boldsymbol{e}} - \boldsymbol{B}\{g(\boldsymbol{x})(1-\alpha)[u_D(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_u) - \hat{u}_{eq}] + f(\boldsymbol{x}) - \hat{f}(\hat{\boldsymbol{x}}\ |\boldsymbol{\theta}_f) + d\}$$

$$\tilde{e}_1 = \boldsymbol{C}^T\tilde{\boldsymbol{e}} \quad (18)$$

where

$$\boldsymbol{\Lambda}_o = \boldsymbol{A} + \boldsymbol{k}_o\boldsymbol{C}^T = \begin{bmatrix} k_n & 1 & 0 & 0 & \cdots & 0 & 0 \\ k_{n-1} & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_2 & 0 & 0 & 0 & \cdots & 0 & 1 \\ k_1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

After $\boldsymbol{k}_o$ being decided, the estimated tracking-error vector ($\hat{\boldsymbol{e}}$) can be solved from (17) and then the estimated state vector ($\hat{\boldsymbol{x}}$) can be found from $\hat{\boldsymbol{x}} = \hat{\boldsymbol{e}} + \boldsymbol{y}_r$.

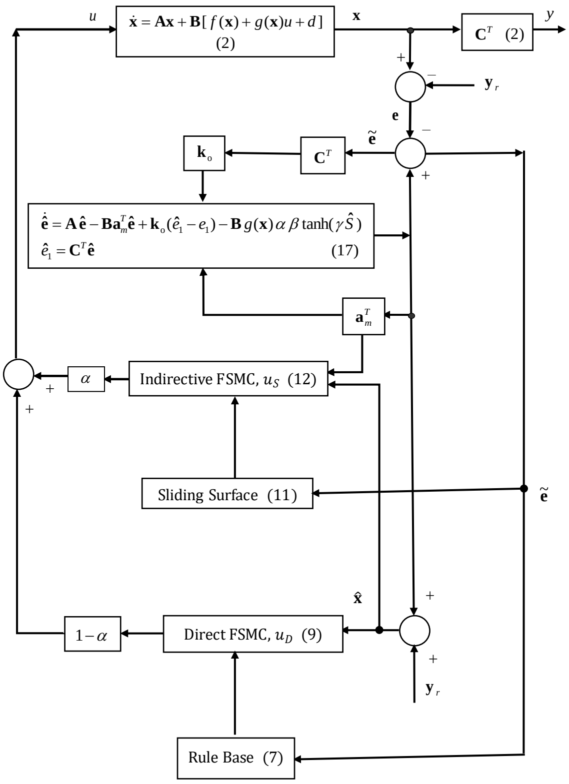


Figure 1. Overall block diagram of the proposed OBHFC.

The overall block diagram of the proposed OBHFC is depicted in Figure 1 and the design process is summarized as follows:

Step 1) Select the observer gain vector $\boldsymbol{k}_o$ such that the characteristic matrix $\boldsymbol{A}+\boldsymbol{k}_o\boldsymbol{C}^T$ is strictly Hurwitz matrix.

Step 2) Specify a positive definite symmetric $n \times n$ matrix $\boldsymbol{Q}$ and solve (20) to obtain a positive definite symmetric $n \times n$ matrix $\boldsymbol{P}$.

Step 3) Choose the suitable sliding surface and a suitable β.

Step 4) Choose the suitable γ in (12) and select a hybrid factor α in (14).

Step 5) Solve the state observer in (17) and calculate $\hat{\boldsymbol{x}} = \hat{\boldsymbol{e}} + \boldsymbol{y}_r$.

Step 6) Define the membership function $A_i^l(\hat{x}_i)$, $B_i^l(\hat{x}_i)$ for $i = 1, \ldots, n$ and compute the fuzzy basis functions $\boldsymbol{\xi}_u(\hat{\boldsymbol{x}})$ and $\boldsymbol{\xi}_f(\hat{\boldsymbol{x}})$. Construct two fuzzy logic systems $u_D(\hat{\boldsymbol{x}} \,|\boldsymbol{\theta}_u)$ and $\hat{f}(\hat{\boldsymbol{x}} \,|\boldsymbol{\theta}_f)$.

Step 7) Go to Step 5).

Application to a Two-Wheeled Self-Balancing Robot

The roll motion and balancing diagram of a practical two-wheeled self-balancing robot is shown in Figure 2. Consider the balance condition, the nonlinear dynamical inverted pendulum system is utilized to derive the balance controller of a two-wheeled self-balancing robot. In this model, a two-wheeled self-balancing robot is approximated by a link and point mass located at the position of the center of gravity. The kinematic equation can be represented as

$$\begin{bmatrix}\dot{\varphi}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\varphi\\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}(\mathrm{f} + \mathrm{gu} + \mathrm{d})$$

$$\mathrm{y} = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\varphi\\ \dot{\varphi}\end{bmatrix} \qquad (19)$$

where

$$f = \frac{(M_c + M_w)g_a\sin(\varphi) - M_c L\sin(\varphi)\cos(\varphi)\dot{\varphi}^2}{L[\frac{4}{3}(M_c + M_w) - M_c\cos^2(\varphi)]} \qquad (20)$$

$$g = \frac{\cos(\varphi)}{L[\frac{4}{3}(M_c + M_w) - M_c\cos^2(\varphi)]} > 0 \qquad (21)$$

φ is the inclined angle of the cant; M_c is the mass of vehicle (mass of two wheels, shaft and gear are excluded); M_w is the mass of two wheels, shaft and gear; g_a is the acceleration due to gravity ($g_a = 9.8$ meter/sec^2); L is the half height of the robot; *d* is the disturbance and u is the control effort.

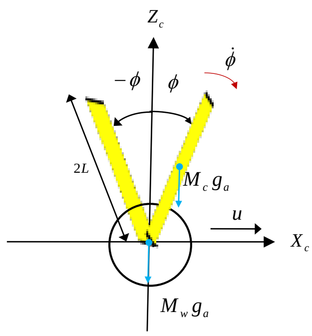


Figure 2. Roll motion of a two-wheeled self-balancing robot

On the balance control approaches, much research has been done in this field. For example, backstepping and fuzzy neural network were used in the cascade adaptive controller [21]; combination of LQR and PID was proposed to overcome the impact of uncertainties [22]; particle swarm optimization (PSO) was employed to get optimum PID controller parameters [23]; neural uncertainty observer was developed to estimate the lumped uncertainty [24]; adaptive output recurrent cerebellar-model-articulation-control was proposed without model information [25] etc. Because the encountered environment of a two-wheeled self-balancing robot may be a bumpy road, the accelerometers and gyroscopes are needed in above mentioned control schemes to provide complete state information. However, the amount of utilized sensors will influence the robot cost, so this study attempts to reduce the hardware cost. To accomplish this motivation, the proposed intelligent OBHFC is extended its application to balance the two-wheeled robot. In the balance controller, the software observer is embedded into microcontroller to replace the hardware gyroscopes.

Numerical Simulation Results

The control interval of the balance control loop is set at 3.8ms, the specifications of the platform are: M_w=1 (kg), M_c=0.1 (kg) and L=0.5 (m). The control objective is to push the inclined angle (φ) of the two-wheeled robot to track the desired trajectory $\varphi_d(t)$, and $\dot{\varphi}$ to track the desired trajectory $\dot{\varphi}_d(t)$; that is, $\boldsymbol{e}$ converges to $\mathbf{0}$. The initial condition is $\varphi = -10$ (degree), $\dot{\varphi} = -2.86$ (degree/sec) and the desired trajectory is $\varphi_d(t) = 0$ (degree), $\dot{\varphi}_d(t) = 0$ (degree/sec). All control parameters are initialized as

$\hat{S}(\hat{\boldsymbol{e}}) = \boldsymbol{a}^T\hat{\boldsymbol{e}}$ where $\hat{\boldsymbol{e}} = [\hat{e}, \dot{\hat{e}}]^T$, $\boldsymbol{a} = [25, 1]^T$;
$\boldsymbol{a}_m = [0, 25]^T$; $\beta = 28$; $\gamma_u = 35$; $\gamma_f = 40$; $\gamma = 7.4$;
$\alpha = 0.6$; $\boldsymbol{k}_o = [-89, -184]^T$; $\boldsymbol{\theta}_u(0) = 1.5$;
$|g| \geq 1.12 = g_L$ $(|\varphi| \leq \pi/6)$;

$\boldsymbol{\theta}_f(0) = 1.5$; $M_{\boldsymbol{\theta}_u} = 40$; $M_{\boldsymbol{\theta}_f} = 40$; $P = \begin{bmatrix} 29 & -14 \\ -14 & 7 \end{bmatrix}$;

$$F = \left| \frac{g_a \sin\varphi - \frac{M_c L \cos(\varphi)\sin(\varphi)\dot{\varphi}^2}{M_w + M_c}}{L\left[\frac{4}{3} - \frac{M_c \cos^2(\varphi)}{M_w + M_c}\right]} \right| \leq \frac{9.8 + \frac{0.025}{1.1}\dot{\varphi}^2}{\frac{2}{3} - \frac{0.05}{1.1}} \approx 15.78 + 0.0366\hat{\dot{\varphi}}^2;$$

Following Gaussian functions are used.

$$A_i^1(\hat{x}_i) = B_i^1(\hat{x}_i) = 1/\{1 + \exp[25(\hat{x}_i + 0.68)]\};$$

$$A_i^2(\hat{x}_i) = B_i^2(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i + \pi/6}{\pi/24}\right)^2\right];$$

$$A_i^3(\hat{x}_i) = B_i^3(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i + \pi/12}{\pi/24}\right)^2\right];$$

$$A_i^4(\hat{x}_i) = B_i^4(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i}{\pi/24}\right)^2\right];$$

$$A_i^5(\hat{x}_i) = B_i^5(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i - \pi/12}{\pi/24}\right)^2\right];$$

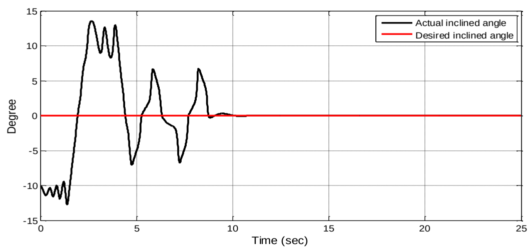
$$A_i^6(\hat{x}_i) = B_i^6(\hat{x}_i) = \exp\left[-\left(\frac{\hat{x}_i - \pi/6}{\pi/24}\right)^2\right];$$

$$A_i^7(\hat{x}_i) = B_i^7(\hat{x}_i) = 1/\{1 + \exp[-25(\hat{x}_i - 0.68)]\};$$

where $i = 1, 2$; $\hat{x}_1 = \hat{\varphi}$; $\hat{x}_2 = \hat{\dot{\varphi}}$. All the parameters are chosen to achieve the best transient control performance considering the requirement of stability. To meet all conditions, the fuzzy linguistic rule base includes 49 rules and three simulation cases including possible external disturbance conditions in bumpy road are addressed as follows:

Case 1: No disturbance.
Case 2: Disturbance = 30 (unit) occurring at 15 (sec) and vanished at 16 (sec) for conventional AFSMC; disturbance = 30 (unit) occurring at 10 (sec) and vanished at 11 (sec) for the proposed OBHFC.
Case 3: Random disturbance.

Now, a conventional AFSMC (complete state information must be provided, no state observer and only direct control effort being considered) under rules 1-49 and the control law in (9) is considered first. The simulated results at Case 1, 2 and 3 are depicted in Figures. 3, 4 and 5, respectively. The robust tracking performance is obvious under the occurrence external disturbance. However, longer response time is needed and the chattering control effort shown in Figures. 3(e), 4(e), 5(e) is serious due to the inappropriate selection of control gain. Then, the proposed OBHFC (state observer being added to provide complete information and two control efforts being considered) under rules 1-49 and the control laws in (5), (9), (12), (14), (17) and (18) are applied to control the balance of a two-wheeled robot. The simulated results at Case 1, 2 and 3 are depicted in Figures. 6, 7 and 8, respectively. From the simulated results, not only favorable tracking response can be obtained under external disturbance but also shorter response time is needed. Compare Figures. 3-5 with Figures. 6-8, the proposed OBHFC is more suitable to balance a two-wheeled robot. As this is a simulation-based study, indeed, the real-world implementation is generally more rigorous/demanding than simulation. In these simulations, the external disturbance $d(t)$ is assumed to have an upper bound (D). Furthermore, if the chattering phenomenon of control effort is too severe, it will also be difficult to implement in practice.



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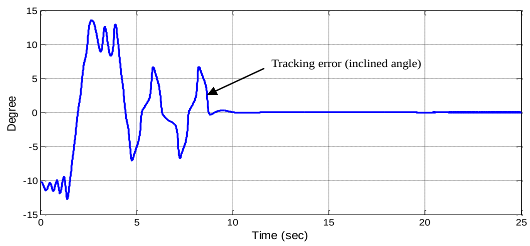

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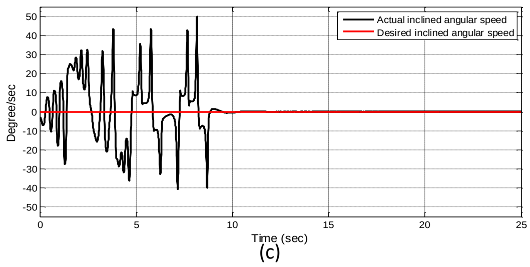

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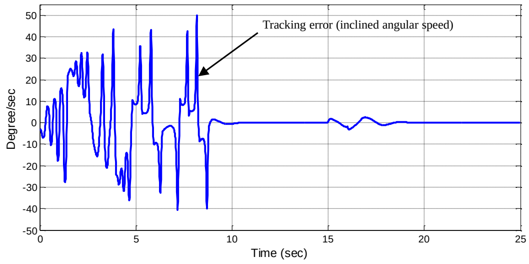

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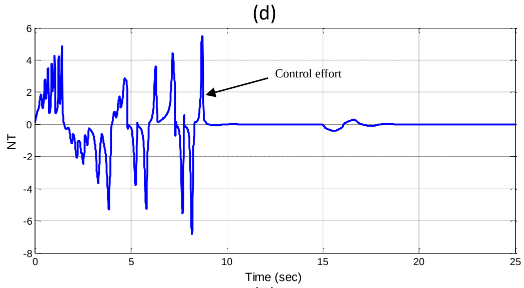
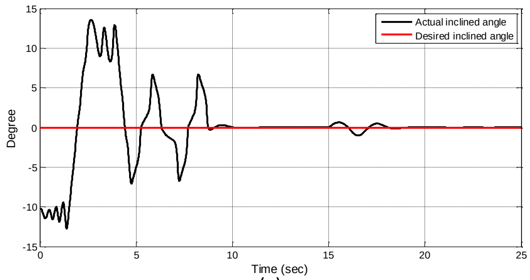


(e)

Figure 3. Simulation results of a conventional AFSMC at Case 1.
(a), (c): tracking responses of inclined angle and inclined angular speed.
(b), (d): tracking error of inclined angle and inclined angular speed.
(e): control effort to balance the robot.



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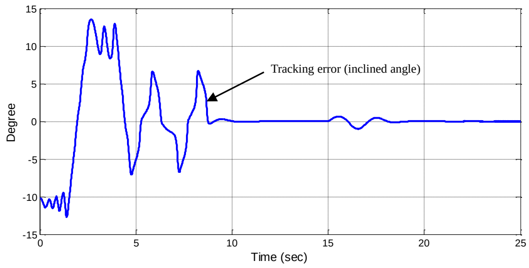

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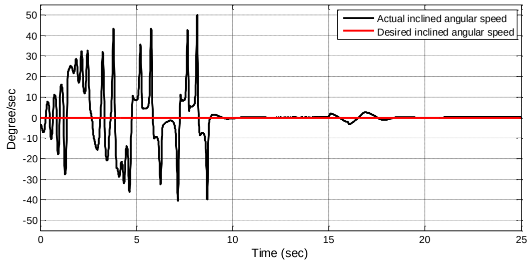

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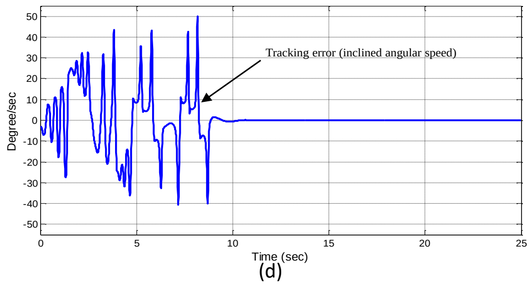

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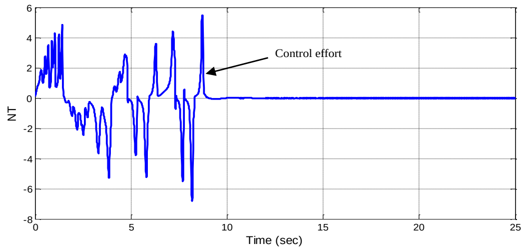
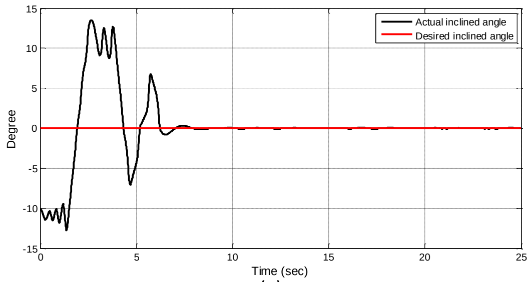


(e)

Figure 4. Simulation results of a conventional AFSMC at Case 2.
(a), (c): tracking responses of inclined angle and inclined angular speed.
(b), (d): tracking error of inclined angle and inclined angular speed.
(e): control effort to balance the robot.



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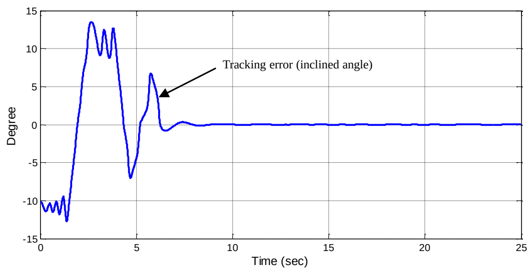

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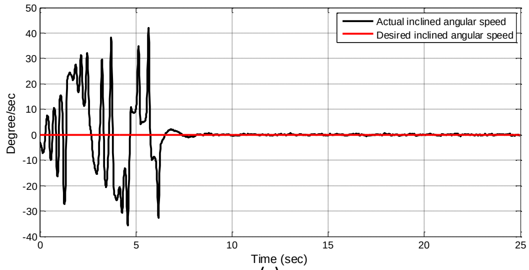

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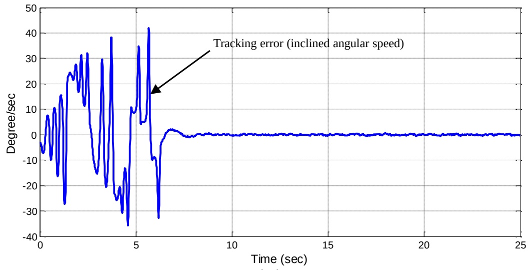

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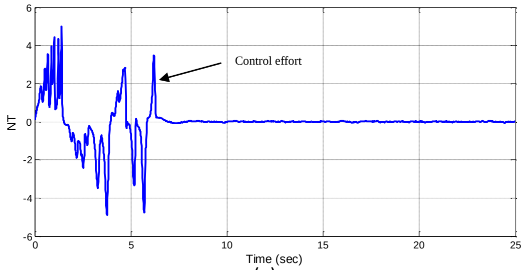


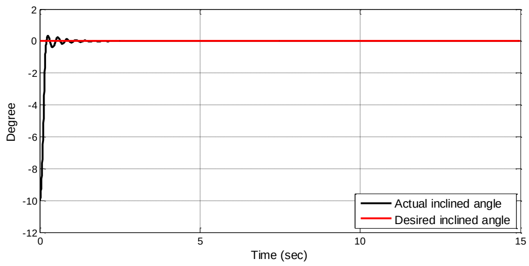
(e)

Figure 5. Simulation results of a conventional AFSMC at Case 3.

(a), (c): tracking responses of inclined angle and inclined angular speed.

(b), (d): tracking error of inclined angle and inclined angular speed.

(e): control effort to balance the robot.



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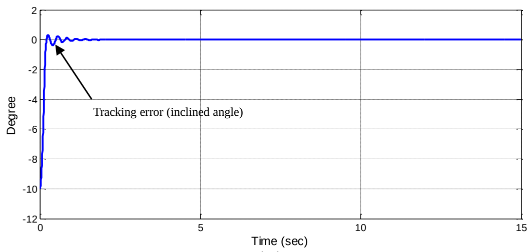

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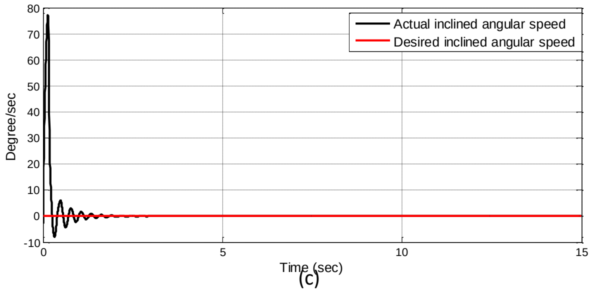

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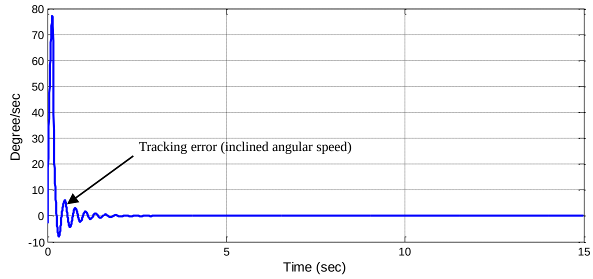

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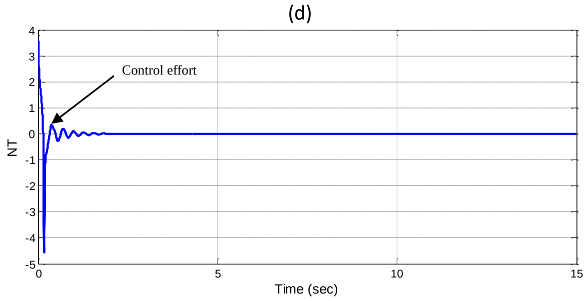


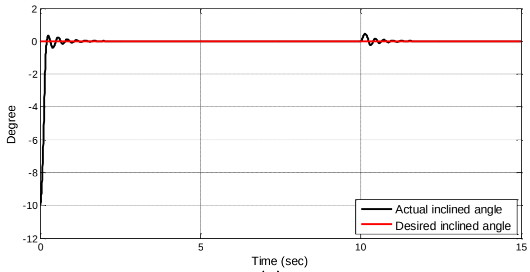
(e)

Figure 6. Simulation results of the proposed OBHFC at Case 1.

(a), (c): tracking responses of inclined angle and inclined angular speed.

(b), (d): tracking error of inclined angle and inclined angular speed.

(e): control effort to balance the robot.



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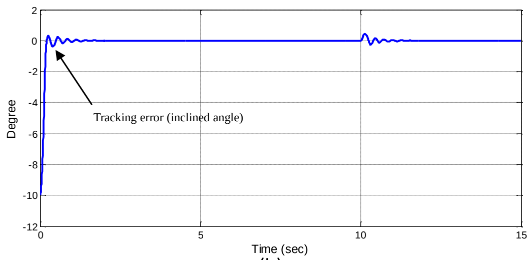

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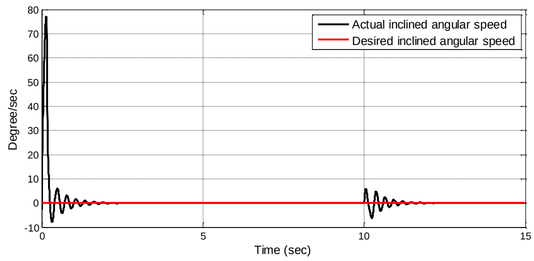

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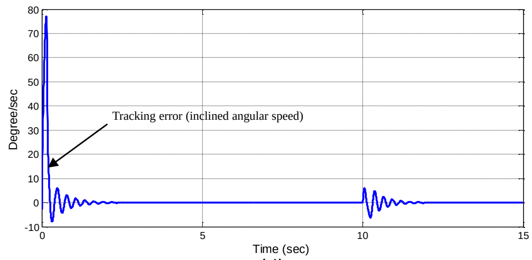

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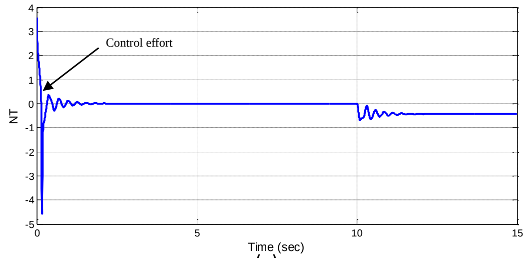


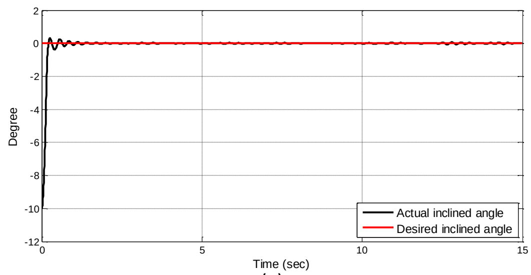
(e)

Figure 7. Simulation results of the proposed OBHFC at Case 2.

(a), (c): tracking responses of inclined angle and inclined angular speed.

(b), (d): tracking error of inclined angle and inclined angular speed.

(e): control effort to balance the robot.



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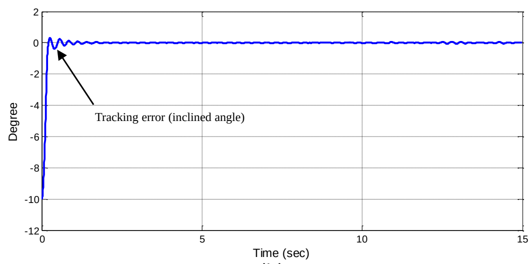

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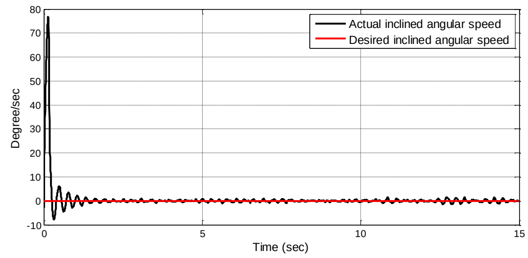

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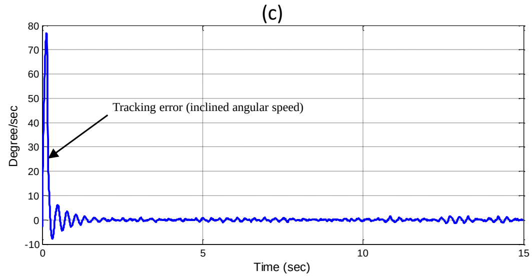

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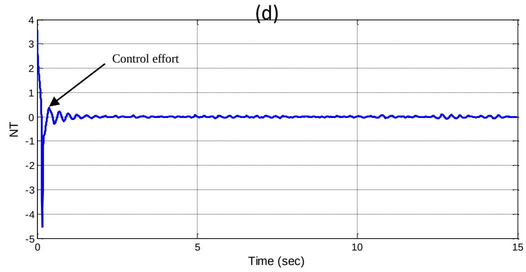

(e)

Figure 8. Simulation results of the proposed OBHFC at Case 3.

(a), (c): tracking responses of inclined angle and inclined angular speed.

(b), (d): tracking error of inclined angle and inclined angular speed.

(e): control effort to balance the robot.

Conclusion

An observer-based hybrid fuzzy controller (OBHFC) is developed in this study. According to the transparency of the plant knowledge and the control information, the hybrid spirit is utilized to build a more powerful adaptive hybrid fuzzy controller (AHFC). To confront the lack of state information, a state observer is added to estimate the tracking error vector, and then the states can be obtained. Utilization of the proposed observer-based output feedback control and adaptation law, the parameters can be tuned in the sense of Lyapunov stability theorem. In order to reduce the robot hardware cost, this study extends the application of the proposed OBHFC to a two-wheeled self-balancing robot. Compared with conventional AFSMC, the proposed OBHFC possesses some features: 1) Better tracking performance and shorter response time can be achieved since hybrid controller is constructed. 2) Lower hardware cost can be achieved since software observer is embedded to replace the hardware gyroscope. 3) Self-tuning property is good even for wicked environment since the adaptation law is added.

Acknowledgment

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